

Maximum Mass of Boson Stars Formed by Self-Interacting Scalar Fields

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Abstract

We make *analytic* derivation for maximum masses of stable boson stars formed by scalar fields with *any* higher order self-interactions and show that those are equivalent to numerical results. It is shown that the contribution of the higher order self-interaction terms to the maximum mass decreases as $(m/M_p)^2$ power.

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I. INTRODUCTION

Though there is no evidence for the presence of a fundamental scalar field, it has played an important role for the study of the earliest stages of the universe [1] [2]. According to this interest, the boson star, which was first discovered theoretically by Kaup [3] and by Ruffini and Bonazzola [4], has become an interesting subject in its own right as well as in the problem of the cosmological missing mass problem. This configuration has been studied in various theories [5] [6] [7] [8]. Their stability [9] [10] and structure [11] have been also investigated in some literatures. (for reviewing the boson star, see Refs. [12] and [13], and other references are cited therein).

Boson stars are formed by soliton-type configurations which are held together simply by their self-generated gravitational field and are only prevented from gravitational collapse by the Heisenberg uncertainty principle. Thus, we can easily calculate the order of radius R_0 , maximum mass M_0^{max} , and energy density ρ_0 of a spherically symmetric stable boson star [3] [4] as

$$R_0 \sim 1/m, \quad M_0^{max} \sim M_p^2/m, \quad \rho_0 \sim M_p^2 m^2, \quad (1.1)$$

where m and M_p are the boson mass and the Planck mass, respectively (we use the unit set by $\hbar = c = 1$). Then, the magnitude of the central value of the scalar field leading to the most massive stable boson stars has the order of the Planck mass, $|\phi| \sim M_p$. In Eq.(1.1), we see that the maximum mass of a boson star is much less than the Chandrasekhar mass, $M_{ch} \sim M_p^3/m^2$. For a bosonic particle with mass 100 GeV, comparing with a neutron star, the maximum mass, radius, and density of a boson star are $10^9 kg \sim 10^{-21} M_{neutron}$, $10^{-18} m \sim 10^{-22} R_{neutron}$, and $10^{63} kg/m^3 \sim 10^{51} \rho_{neutron}$, respectively [13].

The maximum mass of such a stable “mini-boson star” is too small to be a candidate for the missing mass. It is worthy of notice that why a boson star has a much smaller mass than a typical fermionic star is due to the fact that a boson star is only prevented from gravitational collapse by the Heisenberg uncertainty principle instead of the Pauli exclusion principle. In

other words, the Heisenberg uncertainty principle is characterized by a much smaller length scale than that of the Pauli exclusion principle. It has been enhanced by Colpi, Shapiro and Wasserman [5], who considered the boson star formed with self-interacting scalar fields. They have shown that when the order of the coupling constant λ is about unity, the boson star may have the maximum mass comparable to that of a fermionic star.

$$M_{\lambda}^{max} \sim 0.22\Lambda^{1/2}M_p^2/m \sim 0.22\lambda^{1/2}M_{ch}, \quad (1.2)$$

where Λ is a dimensionless quantity defined by $\Lambda \equiv \lambda M_p^2/4\pi m^2$. The origin of this drastic change is that the introduced self-interaction term in matter Lagrangian generates a repulsive force. Thus, the characteristic length scale of the theory becomes large, and the boson star becomes large and massive.

In this paper, we shall make *analytic* derivation for the maximum mass in Eq.(1.2) calculated by numerical analysis. (Analytic derivation of maximum mass of boson star was also appeared in Ref. [14].) Our analytic derivation will be also applied to the boson star formed by scalar fields with *any* higher order self-interactions. Then, a general form of the maximum mass including the self-interaction effect is obtained and it will be shown that the contribution of the higher order self-interaction terms to the maximum mass decreases as $(m/M_p)^2$ power. In order to confirm our analytic calculations, we shall make numerical analysis for the scalar field with the quartic and the sixth order self-interactions¹. In Ref. [13], the authors chose the magnitude of the central value of the scalar field to be the order of the Planck mass, $|\phi| \sim M_p$, even for the self-interacting scalar fields. However, we shall show that when the contribution of the self-interaction becomes important, it has the order of the mass of the scalar field, $|\phi| \sim m\lambda^{-1/2}$ (This relation seems to be singular in the limit of $\lambda \rightarrow 0$. But, our derivation of the relation will be given only in the case that the limit is invalid.) This is consistent with the numerical calculation given in Ref. [5].

¹For the boson star formed by the scalar field with the sixth order self-interaction, see Refs. [15] and [10]

This paper is organized as follows; In Sect.II, the order of the maximum mass of a stable boson star formed by the scalar field with the quartic and the sixth order self-interactions is analytically derived. Then, the formulation is generalized to the scalar field with any higher order self-interactions. Our derivation is confirmed in numerical analysis given in Sect.III. In Sect.IV, we briefly summarize and discuss our results.

II. ANALYTIC DERIVATION FOR MAXIMUM MASSES

In this section we make analytic derivations of the maximum masses of boson stars formed by self-interacting scalar fields. Firstly, this is done for the cases that the matter Lagrangian includes the quartic and the sixth order self-interaction terms. Then, we are going to extend our formulation for any higher order self-interacting bosons.

We begin with the Lagrangian of the scalar field ϕ minimally coupled to gravity given by

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\phi_{;\mu}^*\phi_{;\nu} - \frac{1}{2}m^2|\phi|^2 - \frac{1}{4}\lambda|\phi|^4 - \frac{1}{6}\gamma|\phi|^6, \quad (2.1)$$

where λ and γ are the coupling constants.

As mentioned above, when one is not concerned about the self-interaction, the magnitude of the central value of the scalar field leading to the most massive stable boson stars $|\phi|$ is the order of the Planck mass. When the contribution of the self-interaction becomes important, putting $|\phi|$ is of the order of the Planck mass, however, will give rise to some inconsistencies. This is because the self-interaction generates a repulsive force and the characteristic length scale of the theory becomes larger. To see this, consider only the quartic self-interaction term ($\gamma = 0$) in the Lagrangian (2.1). If one set the order of $|\phi|$ with that of the Planck mass, the energy density including the quartic self-interaction term ρ_λ (Eq.(14) in Ref. [13]) becomes

$$\rho_\lambda \sim m^2|\phi|^2 + \lambda|\phi|^4 \sim m^2M_p^2(1 + \Lambda). \quad (2.2)$$

Comparing Eqs.(1.1) and (2.2), we see that this corresponds to the energy density of a star formed from non-interacting bosons with rescaled mass $m \rightarrow m(1 + \Lambda)^{1/2}$ and the order of the radius of the boson star is given by

$$R_\lambda \sim (1 + \Lambda)^{-1/2}/m. \quad (2.3)$$

Thus, the maximum mass of the boson star becomes

$$M_\lambda^{max} \sim \rho_\lambda R_\lambda^3 \sim (1 + \Lambda)^{-1/2} M_p^2/m \sim \Lambda^{-1/2} M_p^2/m. \quad (2.4)$$

Note that the maximum mass in (2.4) is not equivalent to the numerical result (1.2) given in Ref. [5]. Moreover, ρ_λ in Eq.(2.2) is much greater than ρ_0 in Eq.(1.1) and R_λ in Eq.(2.3) much less than R_0 in Eq.(1.1). It seems to be inconsistent with the fact that the introduced self-interacting potential generates a repulsive force.

Now, turn to the energy density including the quartic self-interaction term. Since we expect from the field equation that the contribution of the self-interaction term is comparable to the mass term in the energy density, $m^2|\phi|^2 \sim \lambda|\phi|^4$, the order of $|\phi|$ becomes

$$\frac{\lambda|\phi|^4}{m^2|\phi|^2} \sim \Lambda \frac{|\phi|^2}{M_p^2} \sim \mathcal{O}(1), \quad |\phi| \sim M_p \Lambda^{-1/2} \sim m \lambda^{-1/2}. \quad (2.5)$$

Note that $|\phi|$ has the order of the scalar mass not the Plank mass. In fact, we can see that in Fig.1 of Ref. [5], the central value of $|\phi|$ decreases with increasing Λ . (The Fig.1 of Ref. [5] was plotted only up to the value $\Lambda = 300$. However, the dimensionless constant Λ is very large, $\Lambda \approx 10^{34} \sim 10^{38}$. For this value, the central density σ_0 in the figure would be very small.) On the other hand, the Eq.(2.5) seems to be singular in the limit of $\lambda \rightarrow 0$. However, as mentioned above, the Eq.(2.5) is only available in the case that the self-interaction term is so large to be comparable to the mass term in the energy density. The energy density becomes

$$\rho_\lambda \sim m^2|\phi|^2 + \lambda|\phi|^4 \sim m^2 M_p^2 \Lambda^{-1}. \quad (2.6)$$

Again, this corresponds to the energy density of a star formed from non-interacting bosons with rescaled mass $m \rightarrow m\Lambda^{-1/2}$. Thus, the radius of the boson star becomes

$$R_\lambda \sim \Lambda^{1/2}/m. \quad (2.7)$$

In Eqs.(2.6) and (2.7), the density and the radius become dilute and large, respectively, and it is consistent with the fact that a repulsive force is involved. Finally, using Eqs.(2.6) and (2.7), we obtain the maximum mass of the boson star equivalent to the numerical result in (1.2) up to constant

$$M_\lambda^{max} \sim \Lambda^{1/2} M_p^2/m \sim \lambda^{1/2} M_{ch}. \quad (2.8)$$

Let us now consider the sixth order self-interacting bosons. Following the above formulation, the central value of the scalar field is given by

$$\frac{\lambda|\phi|^4 + \gamma|\phi|^6}{m^2|\phi|^2} \sim \left(\frac{|\phi|}{M_p}\right)^4 \left[\Lambda \left(\frac{M_p}{|\phi|}\right)^2 + \Gamma \right] \sim \left(\frac{|\phi|}{M_p}\right)^4 (\Lambda^2 + \Gamma) \sim \mathcal{O}(1), \quad (2.9)$$

$$|\phi| \sim M_p(\Lambda^2 + \Gamma)^{-1/4}, \quad (2.10)$$

where $\Gamma \equiv \gamma M_p^4/(4\pi m)^2$. Note that we used the mean field approximation on the second step in Eq.(2.9), i.e., the relation $|\phi| \sim M_p \Lambda^{-1/2}$ given in Eq.(2.5). Thus, the energy density and radius of the boson star are given by

$$\rho_{\lambda\gamma} \sim m^2|\phi|^2 + \lambda|\phi|^4 + \gamma|\phi|^6 \sim m^2 M_p^2 (\Lambda^2 + \Gamma)^{-1/2}, \quad (2.11)$$

$$R_{\lambda\gamma} \sim (\Lambda^2 + \Gamma)^{1/4}/m, \quad (2.12)$$

respectively. As a result, the maximum mass becomes

$$M_{\lambda\gamma}^{max} \sim (\Lambda^2 + \Gamma)^{1/4} M_p^2/m \sim \left(\lambda^2 + \tilde{\gamma} \left(\frac{m}{M_p}\right)^2 \right)^{1/4} M_p^3/m^2 \sim \left(\lambda^2 + \tilde{\gamma} \left(\frac{m}{M_p}\right)^2 \right)^{1/4} M_{ch}, \quad (2.13)$$

where $\tilde{\gamma} \equiv \Gamma(4\pi m)^2/M_p^2 = \gamma M_p^2$. If $\tilde{\gamma} = 0$, the Eq.(2.13) becomes Eq.(1.2). Note that from the field theoretic point of view, $\tilde{\gamma}$ is about unity and the contribution of the sixth order self-interaction to the maximum mass is minor as the order of the inverse square Plank mass comparing with the quartic self-interaction term.

Our above analytic derivations can be easily extended to the case that the matter Lagrangian involves any higher order self-interaction terms. The self-interaction terms can be expressed in terms of dimensionless parameters $\lambda_{(2k+2)}$ as²

$$\lambda_{(4)}|\phi|^4 + \frac{\lambda_{(6)}}{M_p^2}|\phi|^6 + \frac{\lambda_{(8)}}{M_p^4}|\phi|^8 + \dots + \frac{\lambda_{(2n+2)}}{M_p^{2n-2}}|\phi|^{2n+2}. \quad (2.14)$$

These higher order nonrenormalizable terms are naturally expected from the gravity as low energy effective theory and the dimensionful constants are given as the power of $\lambda_{(2k+2)}/M_p^k$.

Then, the magnitude of the central value of the scalar field becomes

$$|\phi|_{(2n+2)} \sim m \left[\sum_{k=1}^n \lambda_{(2k+2)} \left(\frac{m}{|\phi|_{(2n)}} \right)^{2(n-k)} \left(\frac{m}{M_p} \right)^{2k-2} \right]^{-1/2n}, \quad (2.15)$$

where $|\phi|_{(k)}$ is the magnitude of the scalar field considering the order of the self-interaction to be k . The radius and the maximum mass are given by

$$R_{(2n+2)} \sim \frac{M_p}{m^2} \left[\sum_{k=1}^n \lambda_{(2k+2)} \left(\frac{m}{|\phi|_{(2n)}} \right)^{2(n-k)} \left(\frac{m}{M_p} \right)^{2k-2} \right]^{1/2n}, \quad (2.16)$$

$$M_{(2n+2)} \sim \left[\sum_{k=1}^n \lambda_{(2k+2)} \left(\frac{m}{|\phi|_{(2n)}} \right)^{2(n-k)} \left(\frac{m}{M_p} \right)^{2k-2} \right]^{1/2n} M_{ch}, \quad (2.17)$$

respectively. The Eq.(2.17) can be rewritten by

$$M_{(2n+2)} \sim \left[\lambda_{(4)}^n + \lambda_{(6)}^{n/2} \left(\frac{m}{M_p} \right)^2 + \lambda_{(8)}^{n/3} \left(\frac{m}{M_p} \right)^4 + \dots + \lambda_{(2n+2)} \left(\frac{m}{M_p} \right)^{(2n-2)} + \text{cross terms} \right]^{1/2n} M_{ch}. \quad (2.18)$$

As a result, in Eq.(2.18), the contribution of the higher order self-interaction terms to the maximum mass decreases as $(m/M_p)^2$ power.

²The higher order self-interaction potentials arise from effective theories is discussed in Ref. [16].

III. NUMERICAL ANALYSIS FOR MAXIMUM MASSES

In this section, we make numerical analysis to conform maximum masses of stable boson stars calculated in previous section. We only consider up to the sixth order self-interaction term. The equations of motion generated from the Lagrangian (2.1) are given by

$$G_\nu^\mu = 8\pi G T_\nu^\mu, \quad (3.1)$$

$$\nabla^\mu \nabla_\mu \phi - m^2 \phi - \lambda |\phi|^2 \phi - \gamma |\phi|^4 \phi = 0, \quad (3.2)$$

where the energy-momentum tensor $T_{\mu\nu}$ is given by

$$T_\nu^\mu = \frac{1}{2} g^{\mu\nu} (\phi_{;\sigma}^* \phi_{;\nu} + \phi_{;\sigma} \phi_{;\nu}^*) - \frac{1}{2} \delta_\nu^\mu \left[g^{\lambda\sigma} \phi_{;\lambda}^* \phi_{;\sigma} + m^2 |\phi|^2 + \frac{1}{2} \lambda |\phi|^4 + \frac{1}{3} \gamma |\phi|^6 \right]. \quad (3.3)$$

We seek a spherically symmetric, time-independent solution represented by the line element in Schwarzschild coordinates

$$ds^2 = -B(r) dt^2 + A(r) dr^2 + r^2 d\Omega. \quad (3.4)$$

Then, we may choose the ansatz for the scalar field ϕ as

$$\phi(r, t) = \Phi(r) e^{-i\omega t}, \quad (3.5)$$

and the Eqs.(3.1) and (3.2) are written by

$$\frac{A'}{A^2 x} + \frac{1}{x^2} \left(1 - \frac{1}{A}\right) = \left(\frac{\Omega^2}{B} + 1\right) \sigma^2 + \frac{\Lambda}{2} \sigma^4 + \frac{\Gamma}{3} \sigma^6 + \frac{(\sigma')^2}{A}, \quad (3.6)$$

$$\frac{B'}{ABx} - \frac{1}{x^2} \left(1 - \frac{1}{A}\right) = \left(\frac{\Omega^2}{B} - 1\right) \sigma^2 - \frac{\Lambda}{2} \sigma^4 - \frac{\Gamma}{3} \sigma^6 + \frac{(\sigma')^2}{A}, \quad (3.7)$$

$$\sigma'' + \left(\frac{2}{x} + \frac{B'}{2B} - \frac{A'}{2A}\right) \sigma' + A \left[\left(\frac{\Omega^2}{B} - 1\right) \sigma - \Lambda \sigma^3 - \Gamma \sigma^5 \right] = 0, \quad (3.8)$$

where prime denotes d/dx , and $x \equiv mr$, $\sigma \equiv (4\pi G)^{\frac{1}{2}} \Phi$, and $\Omega \equiv \omega/m$. If we write

$$A(x) = \left[1 - \frac{2\mathcal{M}(x)}{x} \right]^{-1}, \quad (3.9)$$

the Eq.(3.6) can be rewritten by

$$\mathcal{M}' = x^2 \left[\frac{1}{2} \left(\frac{\Omega^2}{B} + 1 \right) \sigma^2 + \frac{\Lambda}{4} \sigma^4 + \frac{\Gamma}{6} \sigma^6 + \frac{(\sigma')^2}{2A} \right]. \quad (3.10)$$

The total mass of a boson star is given by

$$M_{\lambda\gamma} = \mathcal{M}(\infty) \left(M_p^2/m \right). \quad (3.11)$$

We require that boundary conditions be $A(0) = 1$, $\sigma(0) = \sigma_0$, $\sigma'(0) = 0$ and $\sigma(\infty) = 0$ so that our solution becomes a nonsingular and asymptotically flat one with finite mass.

First of all, we consider $\Lambda = 0$ case. We can see in Fig.1 that the boson star mass increases with increasing Γ . Explicitly, from Fig.2 plotted the maximum mass of boson star as a function of Γ , we find out that the maximum mass of boson star behaves as

$$M_{\gamma}^{max} \approx 0.24\Gamma^{1/4} M_p^2/m \approx 0.24\tilde{\gamma}^{1/4} \left(\frac{m}{M_p} \right)^{1/2} M_{ch}. \quad (3.12)$$

This result corresponds to the maximum mass in (2.13) with $\Lambda = 0$.

Let us now consider $\Lambda \neq 0$ case. Numerical results of $\mathcal{M}(\infty)$ for $\Lambda = 10, 30, 100$ are shown in Figs.3,4,5. We can see that Γ dependence is not sensible for large value of Λ . From Fig.6 in which we plot the maximum mass of boson star as a function of Γ for several different Λ values. It can be seen that dotted lines representing numerical results approach the solid lines drawing $(0.22^4\Lambda^2 + 0.24^4\Gamma)^{1/4}$. Thus, the maximum mass formed by the self-interacting scalar field is given by

$$\begin{aligned} M_{\lambda\gamma}^{max} &\approx \left(0.22^4\Lambda^2 + 0.24^4\Gamma \right)^{1/4} M_p^2/m \\ &\approx \left(0.22^4\lambda^2 + 0.24^4\tilde{\gamma} \left(\frac{m}{M_p} \right)^2 \right)^{1/4} M_{ch}. \end{aligned} \quad (3.13)$$

This is equivalent to Eq.(2.13) up to constant factors.

Figs.1,3,4,5 show that the numerical noises become noticeable when Γ reaches about 10000. However, the maximum masses of boson stars are not affected by this part of the numerical error.

IV. SUMMARY AND DISCUSSIONS

Boson stars are only prevented from gravitational collapse by the Heisenberg uncertainty principle whose characteristic length scale is enormously small than that of the Pauli exclusion principle. Thus, the radius and mass of a boson star are much less than those of a fermionic star. However, Colpi, Shapiro, and Wasserman [5] have shown that introducing the self-interacting scalar fields, the maximum mass of a stable boson star may be comparable to the Chandrasekhar mass.

In this paper, we have made analytic derivation for the maximum mass and have shown that it is equivalent to their numerical result. In addition, it has been shown that the radius and density of the boson star formed by the self-interacting scalar fields become larger and dilute compared with those of the boson star formed by the scalar fields without self-interaction. The origin of this drastic change is that the introduced self-interaction term in matter Lagrangian generates a repulsive force. Thus, the characteristic length scale of the theory becomes large, and the boson star becomes large and massive. We also generalized our analytic derivation for the boson star formed by scalar fields with any higher order self-interactions. It has been shown that the contribution of the higher order self-interaction terms to the maximum mass decreases as $(m/M_p)^2$ power as expected in the viewpoint of the field theory. Our analytic calculations have been confirmed in numerical analysis in which we considered the scalar field with the quartic and the sixth order self-interactions.

Recently, rotating boson stars have been also considered [17]. This generalization is likely to be natural and it would be interesting to extend our analytic calculation to the rotating boson stars.

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FIGURES

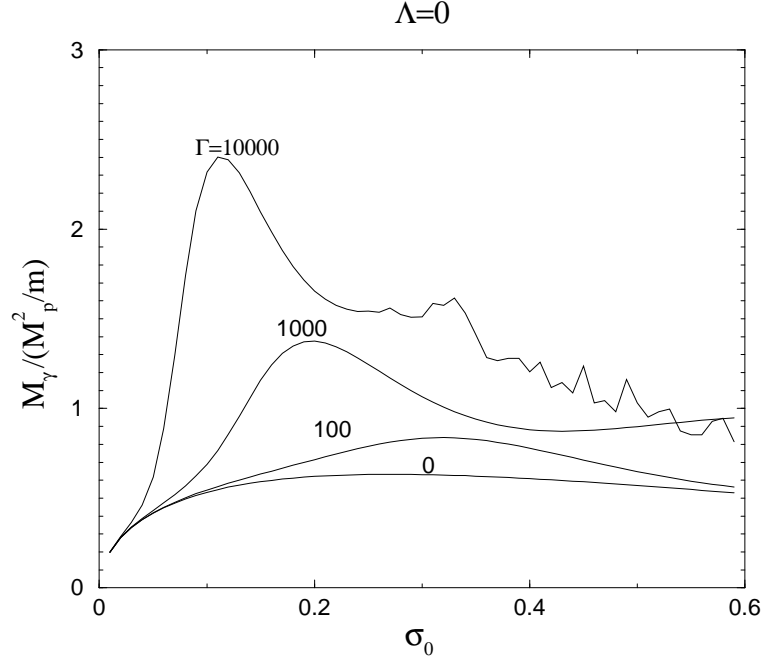


FIG. 1. Boson star mass M_γ as a function of σ_0 when $\Lambda = 0$

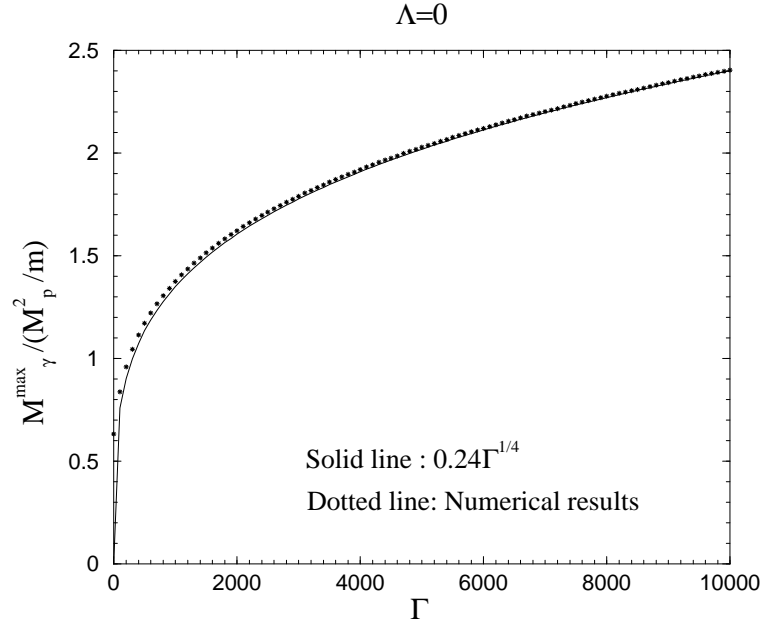


FIG. 2. Maximum mass of boson star as a function of Γ .

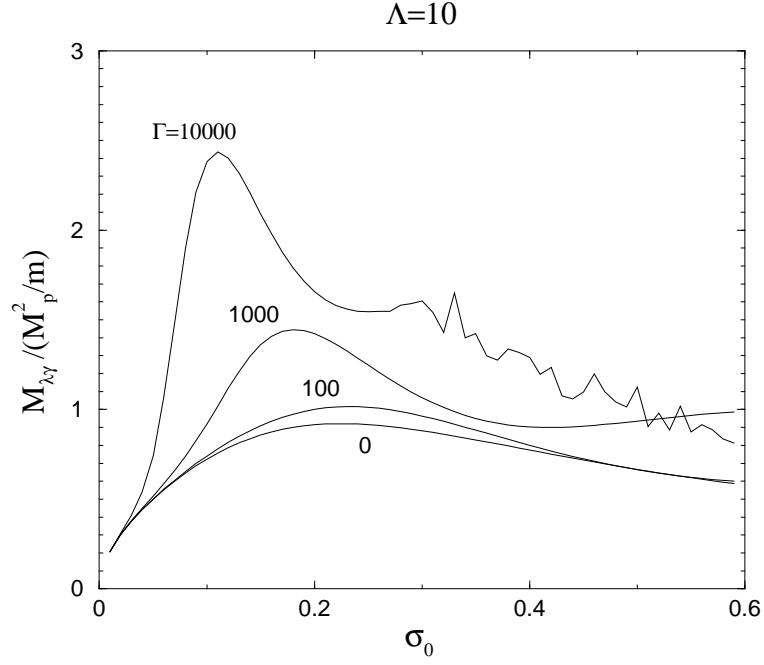


FIG. 3. Boson star mass $M_{\lambda\gamma}$ as a function of σ_0 when $\Lambda = 10$

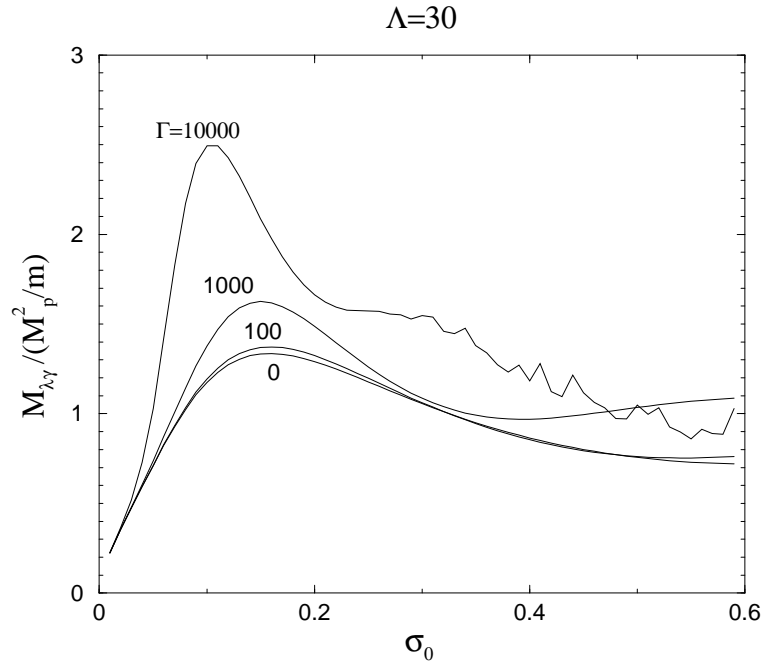


FIG. 4. Boson star mass $M_{\lambda\gamma}$ as a function of σ_0 when $\Lambda = 30$

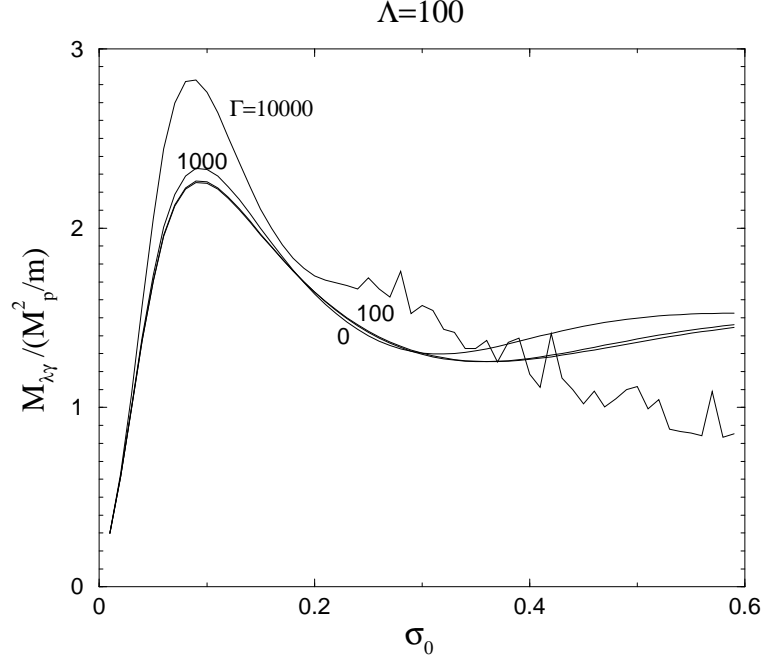


FIG. 5. Boson star mass $M_{\lambda\gamma}$ as a function of σ_0 when $\Lambda = 100$

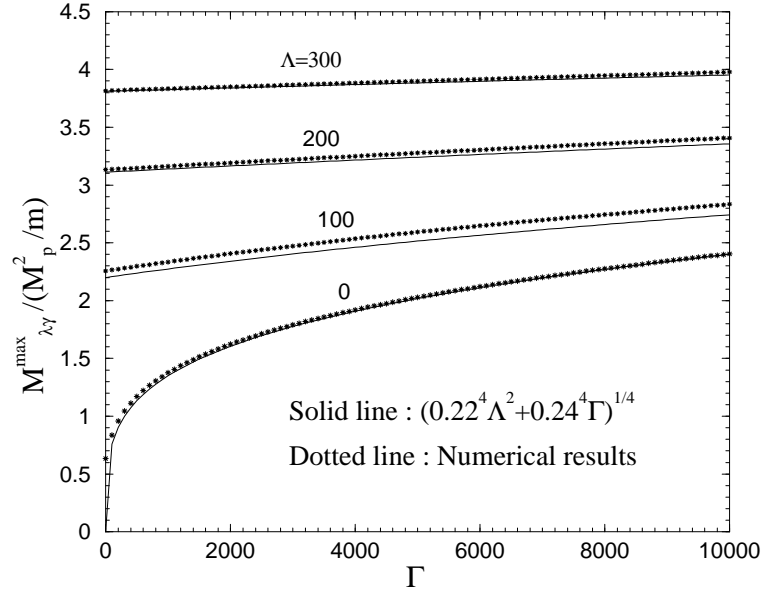


FIG. 6. Maximum mass of boson star as a function of Γ under $\Lambda = 0, 100, 200, 300$.